Beneath Types and Proofs?

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The Problem of the Base Case: no (discernible) connectives

- observational predicates / perceptual types
- Sorites paradox: *heap, small, red, child*, ...
- incremental change and time
  events and *subatomic semantics* (T. Parsons)
  time derived from events/observations (Russell, ...)

→ *denotations* from *parts* of *indices* (in place of connectives)

Proofs outstripping perception

Wang's paradox (a form of Sorites popularized in Dummett 1975)

0 *is small.*
If n is small, then so is $n+1$.
2^{1000} *is* not *small.*

A unary predicate P is *tolerant up to* near_P if

$\text{near}_P(x,y)$ and $P(x)$ implies $P(y)$.

P as temporal proposition (*fluent*)

`Pat was asleep at 11 am.`
Plus or minus a pico-second?

`Pat awoke at noon.`

$\mathbb{R}$ (reals) is *Archimedean*: $\forall \delta > 0$, $\forall x$,

$$\underbrace{\delta + \cdots + \delta}_{n} > x \quad \text{for some integer } n > 0 \ .$$

Where paradox dwells, there NO *type shall be*

Gaifman (2002/10) on tolerant predicates P

- interpret P w.r.t. a *context dependency function* f_P mapping a finite set C (of objects) to a subset $f_P(C)$ of C
- C is *P-feasible* if in C, there are *no* pairs a, b such that $\text{near}_P(a,b)$ and P holds of a but not b
- Claims
  (i) non-feasible contexts do not arise in practice
  (ii) ruling out non-feasible contexts leads to a cumbersome system

Proposal. Focus on contexts that are P-feasible, for all fluents P in a finite set X

$$\boxed{P'}\boxed{P, P'}\boxed{P} \ \in \ \text{Pow}(X)^*$$

Gradience bivalently in strings over intervals

Reify grade between 0 (no) and 1 (yes)
Fluents $\psi(a)$ for incremental change from $\psi(0)$ to $\psi(1)$

	$\psi(a)$ for $0 \le a \le 1$
`Pat swim a mile`	'Pat swim a·[a mile]'
`rain for six hours`	'rain for a·[six hours]'

$\psi_\uparrow \ \approx \ (\exists a < 1)\ \psi(a) \wedge$ Previously $\neg\psi(a)$

$$\boxed{\psi(0)}\boxed{\psi_\uparrow}^+\boxed{\psi(1)}$$

Work with finite samples from $[0,1]$, refining granularity by enlarging samples/observations

$$\boxed{\psi(0)}\boxed{\psi(.5), \psi_\uparrow}\boxed{\psi(1)}$$
$$\boxed{\psi(0)}\boxed{\psi(.3), \psi_\uparrow}\boxed{\psi(.5), \psi_\uparrow}\boxed{\psi(.6), \psi_\uparrow}\boxed{\psi(1)}$$
$$\vdots$$

Refining Prior: proofs as denotations, partially

Prior assumes a linear order $\succ$ on temporal points $t, t' \ldots$

$\mathfrak{A}, t \models p \text{ until } q$ iff $(\exists t' \succ t)\ \mathfrak{A}, t' \models q$ and $\mathfrak{A}, t'' \models p$ for $t \preceq t'' \prec t'$

$$\cdots \boxed{r}\boxed{p,r,t}\boxed{p,r}\boxed{p,r}\boxed{q,p}\boxed{r}\cdots \ \sqsupseteq \ \boxed{p,t}\boxed{p}\boxed{p}\boxed{q}$$
$$\uparrow$$

Proposal. $[\![\varphi]\!]$ is a relation between indices i and denotations d

$i[\![\varphi]\!]d$ iff d is a part of i that $\underbrace{\varphi \text{ is about}}_{\text{is of type } \varphi}$

with φ *true at* each $i \in \text{domain}[\![\varphi]\!]$

$$i \models \varphi \quad \text{iff} \quad (\exists d)\ i[\![\varphi]\!]d$$

The remainder of this talk

§1 X-components (finite set $X \subseteq O$ of observables)
- a system of projections $\pi_X : \text{Pow}(O)^* \to \text{Pow}(X)^*$
- Russell-Wiener-Kamp delimited and Allen relations

§2 Incomplete parts
- partiality (non-bivalence) for denotations, if not indices
- underspecify within string (not just language)
- open-ended processing

§3 Tyranny of logical form
- a notion of part as an underspecified logical connective
- from denotation to index: compositionality in context
- type reduction (unpack syntax)

Grain given by what is observable (boxable)

days in a year $\rightsquigarrow$ months in a year

$$\boxed{\text{Jan,d1}}\boxed{\text{Jan,d2}}\cdots\boxed{\text{Dec,d31}} \quad \overset{\rho_{months}}{\rightsquigarrow} \quad \boxed{\text{Jan}}^{31}\boxed{\text{Feb}}^{28}\cdots\boxed{\text{Dec}}^{31}$$
$$\overset{bc}{\rightsquigarrow} \quad \boxed{\text{Jan}}\boxed{\text{Feb}}\cdots\boxed{\text{Dec}}$$

ρ_X "see only X"

$$\rho_X(\alpha_1\alpha_2\cdots\alpha_n) \ \overset{\text{def}}{=} \ (\alpha_1 \cap X)(\alpha_2 \cap X)\cdots(\alpha_n \cap X)$$

bc "no time without change"
compress α^+ to α [no identical adjacent boxes]

Let bc_X be the sequential composition $\rho_X ; bc$

Intervals over $\mathbb{R}$ and RWK event structures

Over $\mathbb{R}$: a=(1,4), b=(3,9), c=(7,8), d=(11,100)

1 4 8 11

$bc_{\{a,b\}}$(| a | a,b | b | b,c | b | | d |) = | a | a,b | b | |

3 7 9 100

Russell-Wiener-Kamp event structure $\langle E, \bigcirc, \prec \rangle$

| a | a,b | b | | $\rightsquigarrow$ $\langle \{a,b\},\ \{a,b\} \times \{a,b\},\ \emptyset \rangle$ $\rightsquigarrow$ | a,b |

- RWK point is a $\subseteq$-maximal set of $\bigcirc$-overlapping events
- neutralize $\subseteq$-maximality by adding *pre*(b),*post*(a),*post*(b)

(| a | a,b | b |)$_\pm$ = | a,*pre*(b) | a,b | b,*post*(a) | *post*(a),*post*(b) |

$\overset{\rho_{\{a,b\}}}{\rightsquigarrow}$ | a | a,b | b | |

A system of projections

3 conceptions of temporal point based on a set X of events

- as a position in block compressed string over Pow(X)
- as the set of events in X running at that point
- as a $\subseteq$-maximal set of overlapping events in X (RWK)

For finite X of intervals, all 3 agree on $X_\pm$
- Allen relations between a,b from $\{a,b\}_\pm$

For infinite E, form the *inverse limit* of $\{bc_X \mid \text{finite } X \subseteq E\}$

$$\{f \in \prod_{X \in Fin(E)} \mathrm{Pow}(X)^* \mid (\forall X \in Fin(E))(\forall Y \subseteq X)\ f(Y) = bc_Y(f(X))\}$$

- X-component of an event structure is given by RWK on $X_\pm$
- | a | bounds a to the right, but not left

Unpad and π_X

unpad(s) $\overset{\text{def}}{=}$ s with initial or final □'s deleted

unpad(s) = | a | | c | for s $\in$ □* | a | | c | □*

π_X(s) $\overset{\text{def}}{=}$ *unpad*(bc_X(s))

s $\models$ interval(a) iff $\pi_{\{a\}}$(s) $\in$ | a |

| a,b | a | b | $\models$ interval(a)

| a,b | a | b | $\not\models$ interval(b)

For s-intervals a, b,

s $\models a \prec b$ iff $\pi_{\{a,b\}}$(s) $\in$ | a | b | + | a | | b |

$s \models a \bigcirc b$ iff $\pi_{\{a,b\}}(s) \in (|a| + |b| + \epsilon)|a,b|(|a| + |b| + \epsilon)$

iff $s \sqsupseteq |a,b|$ where ...

Containment $\sqsupseteq$ two ways

| a,c | a,b,c | b | $\sqsupseteq$ | a,b |

s $\sqsupseteq$ s$'$ $\overset{\text{def}}{\Leftrightarrow}$ ($\exists$ s$_\circ \in$ factor(s)) s$_\circ \trianglerighteq$ s$'$

- a string s can be shortened from either end

factor(s) $\overset{\text{def}}{=}$ {s$_\circ$ | ($\exists$u, v) s = us$_\circ$v}

| a,b,c | $\in$ factor(| a,c | a,b,c | b |)

- *subsumption* $\trianglerighteq$ between strings of the same length

$\alpha_1 \cdots \alpha_n \trianglerighteq \alpha'_1 \cdots \alpha'_m \overset{\text{def}}{\Leftrightarrow} n = m$ and $\alpha_i \supseteq \alpha'_i$ for $1 \le i \le n$

| a,c | a,b,c | b | $\trianglerighteq$ | c | a,b | b |

Underspecification within a string

$s \models a \bigcirc b$ iff $\pi_{\{a,b\}}(s) \in (|a| + |b| + \epsilon)|a,b|(|a| + |b| + \epsilon)$

iff $s \sqsupseteq |a,b|$

$s \models a \prec b$ iff $\pi_{\{a,b\}}(s) \in |a|b| + |a||b|$

iff $s_\pm \sqsupseteq |a, pre(b)|post(a)|$

$s \models a \subseteq b$ iff $\pi_{\{a,b\}}(s) \in (\epsilon + |b|)|a,b|(\epsilon + |b|)$

iff $\pi_{\{a,b\}_\pm}(s_\pm) \sqsupseteq |pre(a)|a,b|post(a)|$

Finite-state transducers and decidable entailments

FSTs compute *regular relations* between strings

[e.g. $q_0 \overset{\alpha:\alpha'}{\to} q_0$ (for $\alpha' \subseteq \alpha \subseteq X$) computes $\trianglerighteq$]

that constitute the morphisms of a *category* — closed under composition, with domains and codomains regular languages

e.g. $R^{-1}L = \text{domain}(R; \{(s,s) \mid s \in L\})$

$\pi_x = \rho_x; bc; unpad$

Inclusion between regular languages (unlike CFLs) is decidable

Relativize entailment $\vdash$ to constraints C on truth sets domain$[\![\varphi]\!]$

$\varphi \vdash_C \varphi' \overset{\text{def}}{\Leftrightarrow} C \cap \text{domain}[\![\varphi]\!] \subseteq \text{domain}[\![\varphi']\!]$

Information from logical form and denotation to index

In pursuit of information

(Q1) What logical form?
– e.g. main connective?

(Q2) What does the logical form leave out?
– formula/sentence *versus*
occurrence/statement
(utterance including index)

Index as the key to entailments

$$\varphi \vdash_C \varphi' \overset{\text{def}}{\Leftrightarrow} C \cap \text{domain}[\![\varphi]\!] \subseteq \text{domain}[\![\varphi']\!]$$

contra denotations as terms (proofs) in typed lambda calculi.

Revisiting **e**,**t**,**s**

Robin Cooper replaces (unstructured) entities a : **e** by records

$$\textit{record}(a) \quad : \quad [\mathrm{x} : \textit{Ind}]$$

and truth values φ : **t** by record types

$$\textit{record-type}(\varphi) \quad : \quad \textit{RecType}$$

To interpret *record-type*(φ) as a set of situations, and *record*(a) as a situation, he takes a suitable model M as an index for the

$$\text{set } [\![\textit{record-type}(\varphi)]\!]_M \quad \text{and} \quad \text{situation } [\![\textit{record}(a)]\!]_M$$

- interpret = situate in a larger context (index M)
- denotation includes relations to other objects

Inverse limit (above) builds indices (bottom up) from denotations, reversing Carnap/Montague-intension: index $\mapsto$ denotation

Temporal reasoning strung out

Index as a base over which to construct denotation

$$\begin{aligned}
\mathfrak{s} \models (p \text{ until } q)@t \quad &\text{iff} \quad (\exists n \geq 0)\ \mathfrak{s} \sqsupseteq \boxed{t,p}\boxed{p}^n\boxed{q} \\
\mathfrak{A}, t \models \mathsf{G}p \quad &\text{iff} \quad (\forall t' \succ t)\ \mathfrak{A}, t' \models p \\
\mathfrak{s} \models (\mathsf{G}p)@t \quad &\text{iff} \quad \pi_{\{t,p\}_\pm}(\mathfrak{s}_\pm) \sqsupseteq_{\textit{suffix}} \boxed{t}\boxed{p} \\
\mathfrak{s} \sqsupseteq_{\textit{suffix}} \mathfrak{s}' \quad &\overset{\text{def}}{\Leftrightarrow} \quad (\exists u)\ \mathfrak{s} = u\mathfrak{s}'
\end{aligned}$$

Interval temporal logic ($\mathcal{TPL}$, Pratt-Hartmann)

$$\mathcal{A} \models_I \psi \quad \text{iff} \quad \text{str}(\mathcal{A}, I) \in \mathcal{L}(\psi)$$

Inertial φ persists unless forced

$$\begin{aligned}
&\text{forward} && \boxed{\varphi\ } \Rightarrow \boxed{\ \varphi}+\boxed{\mathsf{f}\overline{\varphi}\ } \\
&\text{backward} && \boxed{\ \varphi} \Rightarrow \boxed{\varphi\ }+\boxed{\mathsf{f}\varphi\ }
\end{aligned}$$

Fine trousers for Steedman

My friend Kit Fine has compared the position of the linguist who turns to logic to that of a man in need of trousers who goes to a tailor, only to be told that tailors only make jackets, and that in fact only jackets are necessary, for it is easy to show that jackets are topologically equivalent to trousers.
Such is the authority of logicians that many otherwise decorous persons have found themselves in the position of trying to use jackets as trousers.
When they have complained that jackets don't seem to work very well for the purpose – for example, that the pockets seem to be the wrong way up -- the response has often been impatient.
Sometimes the users have been led to give up on logic entirely and to go off and invent their own knowledge representations.

Sans-culottes

However serviceable these have been, they have often been derided by logicians as outlandish and even indecorous ...
This is a shame, because in the end one's trousers are best made by tailors, and logicians are or ought to be the right people to make knowledge representations.

These notes represent an attempt to bring together the concerns of those who really do need trousers with those who are best fitted to provide them, and to explain why the *sans culottes* on occasion voice needs (or offer alternatives of their own devising, such as "nonmonotonic" logic or "promiscuous" ontologies) that strike logicians as bizarre and uncouth.

Steedman 2005, pp vii-viii

A dozen related papers (available from www.scss.tcd.ie/Tim.Fernando)

- Regular relations for temporal propositions, *Natural Language Engineering* 17(2): 163–184, 2011
- Constructing situations and time, *J. Philos Logic* 40(3):371–396, 2011
- Temporal propositions as vague predicates, Amsterdam Colloquium 2009, pp 143-152
- Situations as indices and as denotations, *L&P* 32(2):185–206, 2009
- Situations in LTL as strings, *Inf & Comp* 207(10): 980–999, 2009
- Branching from inertia worlds, *J. Semantics* 25(3):321–344, 2008
- Observing events and situations in time, *L&P* 30(5):527–550, 2007
- Finite-state temporal projection, CIAA 2006, LNCS 4094, 2006, pp 230–241
- Comic relief for anankastic conditionals, Amsterdam Colloquium 2005, pp 71–76
- Events from temporal logic to regular languages with branching (FG-MOL 2005)
- Inertia in temporal modification, SALT 14, pp 56–73, 2004
- A finite-state approach to events in natural language semantics, *J. Logic & Comp* 14(1):79–92, 2004