Proofs as « explicitations » and floatting types

The development of a proof can be seen as making explicit the constituents of a complex formula and their relations.
Normalisation (cut elimination) satisfies the main ontological requirement of « explicitation »: do not add any new basic entity to the ones referred to by the formula
explicitation=| η-expansion that makes explicit function f keeps η-reduction (that makes function f working) possible and conversely without changes.
Ludics puts the focus on inter-explicitation rather than truth or demonstration.

Seeing ludics as inter-explicitation (1)

Contexts are ways of making things explicit

The duality reversible/irreversible=
the choice of the context does not matter/ the distribution of the contexts does matter
negative connectors: explicitation is ad libitum,
Positive connectors: you have to choose a way of making things explicit

(2) Left/right

In explicitation, we cannot assume that all premises are given on the left
(all the explicitation would be given at once)
We have to select one explicitation perspective on the left
Left : the focus of explicitation,
(**explicitans for explicitanda**)
Right : the different possible associations of the focussed formula with different contexts
= the different possible explicitations of the things to be made explicit
(**explicitanda becoming explicitata**)

(3)Particularisation of the perspectives

The focussed formula on the right is still a general perspective,
The same formula on the left is particularised by different contexts.
When a formula passes from the right to the left, a particularised perspective becomes the general focus for trying still more particularized combinations on the left
Progress toward made explicit particularisation

(4)inter-explicitation

Negative rule

When “You”offers to “I” a range of contextualisation (Right) from a focal perspective (Left),

“I” can choose **one** of them (**ad libitum**),

and then “You” has to explore the chosen perspective

Positive rule

When « I » chooses **one** contextualisation (Right)

as a focal perspective of explicitation (Left) (irreversible move, dependent on the choice of context),

then “I” has to explore **each** of the combinations of this perspective with the sub-contexts.

Admitting different perspectives implies to accept to consider any of them.

Choice of a perspective implies to develop its explicitation

These rules ensure the fan out (diversity) and the depth of explicitation

(5)Dialogue

Negative rule at first, introduction of the dialogue, context remains the same
Then positive : Focalisation on a perspective, combination with its contexts,
Cut
Confirmation of the coordination on the same way of contextualisation:

(6) From explicitation to validation

One explicitation cannot ensure its validation
(other explicitations are possible)
Interaction: « I » (resp.« You ») considers F as a perspective of explicitation,
« You » (resp. « I » considers F as an element to make explicit in a variety of contexts
Partial validation: two formulas are in symmetrical position,
Cut: stationarity of the interactive meaning
Daimon or failure of interaction for one perspective
=Validation of the symmetrical perspective

Relation with qualification and predication

A particular colour of an object is the result of the meeting between a perspective, its explicitation through the duality of the structure of the visual system and the structure of the environment (proof and counter-proof)
Bringing forth its emergence from this global system as a stable colour, (stationarity or cut)
That is itself a perspective for other combinations (forms, brightness, etc.)
that constitute a **qualified object**
Perception is a “dualogue” with environment

Dialogue and dualogue

Dualogue with environment: simple stationarity
Dialogue: bi-stationarity:
For partner 1,the explicitation perspective of partner 2 becomes itself an element to make explicit by contextualisation,
And symmetrically for partner 2:
double dualogue

Fundamental interactive types (as ontological types?)

pointers (the anchorage of focalisation) = loci= e
Validation or test = truth = t
i for interaction = local cuts plus dual rules
i implies duality, shift between the dual interactive rules,
exchanging the roles of explicitating perspective and making explicit formula by its contexts

Types and degrees

We cannot take a formula *at the same time* as a perspective of explicitation and as an element of the system of contextualisation that specifies the situation
successive shifts between these dual functions are necessary
Each new shift goes deeper in the specification, so that **degrees of particularisation** can be defined inside the type i (interaction)
But their definition is *path dependent*

Reductions

Validation is reduction to type t
But cuts are also local reductions (to type i, with degrees)
And in a logic with cut elimination,
since proving is reduced to permutations of elementary formulas,
normalisation can be considered as a a reduction to loci (the most particular residual adresses, each disjoint from the others)= e

Floatting types ?

The use of an expression depends on its contexts
The type of use of a word of a grammatical category can differ in accordance with the context (understood as the successive choices among possible combinations of an expression with other ones; extendable to the whole dialog and more)
and to the depth and fan out of explicitation
Ex: an adjective can play the role of a predicate (‘small Bill’), or the role of modifying a complex noun as an adverb (‘stationary formal relation’)

Floatting types ?

Basically indexicals and proper names: pointers = e
Common nouns imply interactions= i
(e,i,….), but not necessarily (e,t)
(e) can be demultiplied (intransitive to transitive verb)
A complex noun phrase can be used just as a pointer (….)e
An adverb, instead of ((e,t),(e,t)), a modifier of the validation of a validable compound can be used as (e,i)i (a specifier of a meaning in order to keep on the interaction in the dialog)

Limit cases

Negations : not only Geach type changing (adding the same context to different types), but also changing the mode of duality of interaction (“I do not know p” vs. “I know that not p”)

Irony: forced regress of type t to type i ?

Would be exponents or superscripts useful in order to indicate the degree of specification of the type in the dialog?

Most of the time the steps of the real dialog are not sufficient to specify it (floatting degrees)

What about reduction of floatting types? Our ideal is reduction to type t,
normal dialogues are reducible to type i,
and there is still a possible reduction to e, pointing just to the expressions that have been used!

This talk is hoped to have been of type i….